Iso-Spectral Potentials and Inflationary Quantum Cosmology

A. García,[1] W. Guzmán,[1] M. Sabido,[1] and J. Socorro[1,2]

Using the factorization approach of quantum mechanics, we obtain a family of isospectral scalar potentials for power law inflationary cosmology. The construction is based on a scattering Wheeler-DeWitt solution. These iso-potentials have new features, they give a mechanism to end inflation, as well as the possibility to have new inflationary epochs. The procedure can be extended to other cosmological models.

1. INTRODUCTION

One of the most active areas of research nowadays is inflationary cosmology, this theoretical framework solves many classical problems of Standard Big Bang Cosmology. Recently, observations have confirmed its predictions of a flat universe with a nearly scale invariant perturbations spectrum. The idea is to introduce a scalar field (spin-0 boson) and a scalar potential $V(\phi)$ which encodes in itself the (non-gravitational) self-interactions among the scalar particles. This type of models, have also been used within the so called canonical Quantum Cosmology (QC) formalism, which deals with the early epoch of the cosmos. Scalar fields act as matter sources, and then play an important role in determining the evolution of the early universe, where the quantum fluctuations are the seeds for structure formation.

Moreover, these models have appeared in String Theory, in particular in connection to the so called string theory landscape as well as in the study of tachyon dynamics. For the String Theory landscape (Douglas, 2003; Kobakhidze and Mersini-Houghton, 2004; Susskind, 2003), the scalar potential $V(\phi)$ is usually thought as having many valleys, which represent the different vacua solutions, the

---

[1] Instituto de Física de la Universidad de Guanajuato, A.P. E-143, C.P. 37150, León, Guanajuato, México.
[2] To whom correspondence should be addressed at; e-mail: socorro@fisica.ugto.mx.

hope is that the statistics of these vacua could explain, for example, the smallness of the cosmological constant (the simplest candidate for dark energy). For tachyon dynamics in the unstable D-brane scenario, the scalar potential for the tachyon effective action around the minimum of the potential has the form $V(\phi) = e^{-\alpha\phi/2}$ (Sen, 2002, 2003). This leads to the study of tachyon driven cosmology (Garcia-Compean *et al.*, 2005; Gorini *et al.*, 2004).

On another front, the study of eigenvalue problems associated with second-order differential operators found a renewed interest with the application of the factorization technique and its generalizations (Cooper *et al.*, 1995; Fernández, 1984; Gelfand and Levitan, 1955; Mielnik, 1984; Nieto, 1984). SUSY-QM may be considered an equivalent formulation of the Darboux transformation method, which is well-known in mathematics from the original paper of Darboux (Darboux, 1982; Ince, 1926). An essential ingredient is a differential operator (Bagrov and Samsonov, 1995) which intertwines two hamiltonians and relates their eigenfunctions. When this approach is applied in quantum theory it allows to generate a family of exactly solvable local potential starting with a given exactly solvable one (Cooper *et al.*, 1995). In nonrelativistic one-dimensional supersymmetric quantum mechanics, the factorization technique was applied to the $q = 0$ factor ordered WDW equation corresponding to the FRW cosmological models without matter field (Rosu and Socorro, 1998), where a one-parameter class of strictly isospectral cosmological FRW solutions was exactly found, representing the wave functions of the universe for that case, also iso-spectral solutions for a one-parameter family of closed, radiation-filled FRW quantum universe, and for a perfect fluid with barotropic state equation and cosmological constant term, for any factor ordering were found (Rosu and Socorro, 1996; Socorro *et al.*, 2003). In this formalism, the family of iso-potentials and wave functions are build with respect to a parameter γ_i for which we choose the domain $[0, \infty]$. The shape of the wave function in the corresponding coordinates is obtained via the "evolution" of the iso-wave function when this parameter tends to ∞.

The main purpose of this paper is to apply the Darboux transformation method to obtain a family of iso-potentials, to the potential $V(\phi) = e^{-\alpha\phi/2}$, which appears in inflationary cosmology. To reach this goal, we shall make use of the strictly isospectral scheme based on the general Riccati solution (Cooper *et al.*, 1995; Fernández, 1984; Mielnik, 1984; Nieto, 1984), which is also known as the double Darboux method. This scheme has been applied from classical and quantum physics (Mielnik, 1984) to relativistic models (Samsonov and Suzko, 2003). This technique has been known for a decade in one-dimensional supersymmetric quantum mechanics (SUSY-QM) and usually requires nodeless, normalizable states of a Schrödinger-like equation. However, Pappademos *et al.* (1993) showed that the strictly isospectral construction can also be performed on non-normalizable states. The resulting potentials have interesting features, in particular they solve the one problem the exponential potential has, the lack of a mechanism to end inflation.

This work is organized as follows. In Section 2 we present the classical action with the corresponding contributions, this action includes a gravitational part S_g, and S_ϕ for the scalar field; also we present the standard quantum scheme with the quantum solution, which plays an important role in the isospectral solutions. In Section 3 we review the factorization approach of (Susy-QM) and apply it to the inflationary model. Finally Section 4 is devoted to conclusions and outlook.

2. THE STANDARD QUANTUM SCHEME

We start with the line element for a homogeneous and isotropic universe, the so called Friedmann-Robertson-Walker (FRW) metric, in the form

$$ds^2 = -N^2(t)dt^2 + e^{2\alpha(t)}\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \qquad (1)$$

where $a(t) = e^{\alpha(t)}$ is the scale factor, $N(t)$ is the lapse function, and κ is the curvature constant that takes the values $0, +1, -1$, which correspond to a flat, closed or open universe, respectively.

The effective action we will be working, is (Guzmán *et al.*, 2005)

$$S_{\text{tot}} = S_g + S_\phi = \int dx^4 \sqrt{-g}\left[R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V_0 e^{-\frac{\lambda}{\sqrt{12}}\phi}\right], \qquad (2)$$

ϕ is a scalar field endowed with a scalar potential $V(\phi) = V_0 e^{-\frac{\lambda}{\sqrt{12}}\phi}$.

The Lagrangian for a FRW cosmological model is

$$\mathcal{L} = e^{3\alpha}\left[6\frac{\dot{\alpha}^2}{N} - \frac{1}{2}\frac{\dot{\phi}^2}{N} + N\left(V(\phi) - 6\kappa e^{-2\alpha}\right)\right], \qquad (3)$$

At this point, we consider a flat universe ($\kappa = 0$)

$$\mathcal{L} = e^{3\alpha}\left[6\frac{\dot{\alpha}^2}{N} - \frac{1}{2}\frac{\dot{\phi}^2}{N} + NV(\phi)\right], \qquad (4)$$

The canonical momenta are found to be

$$\Pi_\alpha = \frac{\partial\mathcal{L}}{\partial\dot{\alpha}} = 12e^{3\alpha}\frac{\dot{\alpha}}{N}, \qquad \dot{\alpha} = \frac{N}{12}e^{-3\alpha}\Pi_\alpha, \qquad (5a)$$

$$\Pi_\phi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = -e^{3\alpha}\frac{\dot{\phi}}{N}, \qquad \dot{\phi} = -Ne^{-3\alpha}\Pi_\phi. \qquad (5b)$$

We are now in position to write the corresponding canonical Hamiltonian (Ryan, 1972)

$$\mathcal{L}_{\text{canonical}} = \Pi_\alpha\dot{\alpha} + \Pi_\phi\dot{\phi} - N\mathcal{H}, \qquad (6)$$

where $\mathcal{H}$ is the classical Hamiltonian function, having the following structure

$$\mathcal{H} = \frac{1}{24} e^{-3\alpha} \left[\Pi_\alpha^2 - 12\Pi_\phi^2 - 24e^{6\alpha} V(\phi) \right] , \tag{7}$$

and performing the variation of (6) with respect to N, $\partial\mathcal{L}/\partial N = 0$, implies the well-known result $\mathcal{H} = 0$. The Wheeler-DeWitt (WDW) equation for this model is achieved by replacing Π_{q^μ} by $-i\partial_{q^\mu}$ in Eq. (7); here $q^\mu = (\alpha, \phi)$.

Under a particular factor ordering the WDW reads

$$\hat{H} = \frac{e^{-3\alpha}}{24} \left[-\frac{\partial^2}{\partial\alpha^2} + 12\frac{\partial^2}{\partial\phi^2} - 24e^{6\alpha} V(\phi) \right] \Psi = 0. \tag{8}$$

or

$$\Box \Psi - 24e^{6\alpha} \tilde{V}(\phi)\Psi = 0 , \tag{9}$$

with $\tilde{\phi} = \frac{\phi}{\sqrt{12}}$, Ψ is called the wave function of the universe, $\Box \equiv -\partial_\alpha^2 + \partial_{\tilde{\phi}}^2$ is the two dimensional d'Alambertian operator in the q^μ coordinates. From now on we fix the potential to $V(\phi) = e^{-\lambda\tilde{\phi}}$. Applying the factorization method in these variables is technically cumbersome, this can be simplified if we make the following coordinates transformation,

$$x = 6\alpha - \lambda\tilde{\phi}, \qquad y = \alpha - \frac{6}{\lambda}\tilde{\phi}, \tag{10}$$

the WDW Eq. (9) takes the form

$$\frac{\partial^2\Psi}{\partial x^2} - \frac{1}{\lambda^2}\frac{\partial^2\Psi}{\partial y^2} - \frac{24V_0}{\lambda^2 - 36}e^x\Psi = 0 \tag{11}$$

and by separation variables, $\Psi = X(x)Y(\tilde{y})$ with $\tilde{y} = \lambda y$, we obtain the set of differential equation for the functions X and Y

$$\frac{d^2X}{dx^2} + \left(-\beta e^x + \frac{\eta^2}{4} \right) X = 0,$$

$$\frac{d^2Y}{d\tilde{y}^2} + \frac{\eta^2}{4}Y = 0, \tag{12}$$

where

$$\beta = \frac{24V_0}{\lambda^2 - 36}, \tag{13}$$

we choose for simplicity $\frac{\eta^2}{4}$ as a separation constant. The solutions for these equations are

$$X(x) = Z_{\pm i\eta}\left(\pm 2i\sqrt{\beta}e^{x/2} \right), \tag{14}$$

$$Y(\tilde{y}) = A_0 e^{i\frac{\eta\lambda}{2}\tilde{y}} + A_1 e^{-i\frac{\eta\lambda}{2}\tilde{y}}, \tag{15}$$

with $Z_{\pm i\eta}$ are generic Bessel Functions with pure imaginary order, the wave function is

$$\Psi_\eta(x, \tilde{y}) = e^{\pm i \frac{\eta\lambda}{2} \tilde{y}} Z_{i\eta} \left(\pm 2i\sqrt{\beta} e^{x/2}\right). \tag{16}$$

Since these solutions have the dependence in the parameter η, the general solution can be put as

$$\Psi_{\text{gen}} = \int G(\eta)\Psi_\eta d\eta, \tag{17}$$

where $G(\eta)$ represents a weigh function.

The selection of the value of λ in (13), gives the structure for the $Z_{\pm i\eta}$, for $\lambda > 6$, we have the modified Bessel function, and for $0 < \lambda < 6$, $Z_{\pm i\eta}$ become the ordinary Bessel function. We are now in a position to form a normalizable *Gaussian state* as a superposition of the eigenfunctions (16). With this in mind a *wave packet* can be constructed (Kiefer, 1988, 1990).

We can have different solutions that depend on the value of the parameter λ. For $\lambda > 6$, the *wave packet* can be constructed using the modified Bessel function (see Gradshteyn and Ryzhik, 1980)

$$\Psi(x, \tilde{y}) = \int_0^\infty \cos\left(\frac{\eta\lambda}{2}\tilde{y}\right) K_{i\eta}\left(\pm 2\sqrt{\beta} e^{x/2}\right) d\eta$$
$$= \frac{\pi}{2}\text{Exp}\left[-2\sqrt{\beta} e^{x/2} \cosh\left(\frac{\lambda}{2}\tilde{y}\right)\right]. \tag{18}$$

In the range $0 < \lambda < 6$ (which includes the inflation), we can also construct a *wave packet*, but this time using the ordinary Bessel functions

$$\Psi(x, \tilde{y}) = \int_{-\infty}^\infty \frac{e^{\pi x/2}}{\sinh(\pi x)} \cos\left(\frac{\eta\lambda}{2}\tilde{y}\right) J_{i\eta}\left(\pm 2\sqrt{\beta} e^{x/2}\right) d\eta$$
$$= -i\text{Exp}\left[i2\sqrt{\beta} e^{x/2} \cosh\left(\frac{\lambda}{2}\tilde{y}\right)\right]. \tag{19}$$

We have been using the new variables x and $\tilde{y}$. Let us now extract some information from the semiclassical behavior as a check of our quantum model. The classical solutions can be obtained using the semiclassical analysis (WKB-like method). For this, one considers the ansatz on the wave function

$$\Psi(\alpha, \phi) = e^{-S}, \tag{20}$$

and the conditions

$$\left(\frac{\partial S}{\partial a}\right)^2 \gg \left|\frac{\partial^2 S}{\partial a^2}\right|, \qquad \left(\frac{\partial S}{\partial \phi}\right)^2 \gg \left|\frac{\partial^2 S}{\partial \tilde{\phi}^2}\right|, \tag{21}$$

from this, the Einstein-Hamilton-Jacobi equation (EHJ) is obtained, and Eq. (9) reads

$$\left(\frac{\partial S}{\partial \alpha}\right)^2 - \left(\frac{\partial S}{\partial \tilde{\phi}}\right)^2 - 24V_0 e^{6\alpha - \lambda\phi} = 0, \tag{22}$$

The same equation is recovered directly when we introduce the transformation on the canonical momentas

$$\Pi_{q^\mu} \to \frac{\partial S}{\partial q^\mu}, \tag{23}$$

in Eq. (7), in the new coordinates x and y, this equation takes the form

$$\left(\frac{\partial S}{\partial x}\right)^2 - \left(\frac{\partial S}{\partial \tilde{y}}\right)^2 - \beta e^{-x} = 0. \tag{24}$$

and choosing $S = S_x S_y$ we obtain the following solutions

$$S_x = \pm\frac{2}{\sqrt{\beta}\mu} e^{-x/2},$$

$$S_{\tilde{y}} = \mu = cte. \tag{25}$$

so we get,

$$S(x, \tilde{y}) = \pm 2\sqrt{\beta} e^{-x/2}. \tag{26}$$

Equation (23) in the new variables become

$$\Pi_\alpha = \pm\frac{6}{\sqrt{\beta}} e^{-x/2} = \pm\frac{6}{\sqrt{\beta}} e^{3\alpha - \frac{\lambda}{2}\phi},$$

$$\Pi_\phi = \mp\frac{\lambda}{\sqrt{\beta}} e^{-x/2} = \pm\frac{\lambda}{\sqrt{\beta}} e^{3\alpha - \frac{\lambda}{2}\phi}, \tag{27}$$

The classical behaviour is considered solving the relations between (27) and Eqs. (5a,5b), obtaining the relation for the the scale factor and the scalar field

$$a = a_0 e^{\frac{1}{2\lambda}\tilde{\phi}}, \tag{28}$$

the corresponding time behaviour $(d\tau = Ndt)$,

$$a = a_0 \tau^{\frac{2}{\lambda^2}},$$

$$\tilde{\phi} = \frac{2}{\lambda} Ln\left(\frac{\lambda^2}{4\sqrt{3\beta}}\tau + \tilde{\phi}_0\right), \tag{29}$$

having an inflationary scenario for the scale factor, with increasing power law when $\lambda < \sqrt{2}$. This is the result from the classical approach (Chimento and Jakubi,1996). Unfortunately this potential has a serious drawback, once we fix the value of λ to for inflation, there is no way to end it.

3. SUPERSYMMETRIC FACTORIZATION SCHEME

As already mentioned, the goal of this paper is to use the factorization approach of supersymmetric quantum mechanics to obtain the family of isoespectral potentials to $V(\phi) = e^{-\lambda\tilde\phi}$, an see if these potentials lead to new physics.

If a quantum system is characterized by Hamiltonian H with eigenvalues ν, we may ask if there exists another Hamiltonian (i.e. different potential) which has the same spectrum. This is an interesting question because we may not be able to distinguish one system from the other by making them physically equivalent. With this in mind, we will begin by reviewing the factorization approach.

Lets start with the hamiltonian that appears in the study of scalar fields interacting with gravitation Eqs. (11, 12), written in generic form as

$$-\frac{d^2 P_i}{dq_i^2} + V_i(q_i)P_i = E_i P_i, \quad P_i = (X, Y), \quad q_i = (x, y),$$

$$E_i = \begin{cases} \frac{\nu^2}{4\mu^2} \\ \frac{B^2\nu^2}{\mu^2} \end{cases},$$

$$V_i = \begin{cases} -\frac{6}{\mu^2}e^x \\ 0 \end{cases}. \tag{30}$$

It is easy to show that the first order differential operators

$$\mathcal{A}_i^+ = -\frac{d}{dq^i} + W_i(q^i), \tag{31}$$

$$\mathcal{A}_i^- = \frac{d}{dq^i} + W_i(q^i), \tag{32}$$

factorize the hamiltonian, l.h.s. of Eq. (30)(here W_i plays the role of a superpotential function) as

$$H_i^+ - E_i = \mathcal{A}_i^+ \mathcal{A}_i^- \qquad i = 1, 2. \tag{33}$$

The potential term $V_i(q^\mu)$ is related to the superpotential function $W_i(q^\mu)$ via the Riccati equation[3]

$$V_i(q^i) - E_i = W_i^2 - \frac{dW_i}{dq^i}, \qquad i = 1, 2 \tag{34}$$

Making the transformation

$$W_i = -\frac{u_i'}{u_i}, \quad i = 1, 2 \tag{35}$$

---

[3] We shall call V_i^+ to V_i in (14) and (15).

where the $\prime$ means $\frac{d}{dq^i}$, (34) is transformed into the original hamiltonians applied to the functions u_i that correspond to the solutions (14,15), this implies that once we have a solution to the original Schrödinger like Eq. (30) the superpotential function can be constructed. In this factorization scheme, V_i^- is the partner superpotential of V_i^+, and can be calculated by performing the product

$$H_i^- - E_i = \mathcal{A}_i^- \mathcal{A}_i^+, \qquad H^- f_i(q^i) = E_i f_i(q^i), \tag{36}$$

where f_i is the wave function related to the hamiltonian H_i^-. Then, the isospectral potential to $V_i^+(q^i)$ is

$$V_i^-(q^i) - E_i = W_i^2 + W_i'. \tag{37}$$

Using (34), we get the functional relation between V_i^- and V_i^+, being

$$V_i^-(q^i) = V_i^+(q^i) + 2W_i'. \tag{38}$$

However, this is not the most general solution as will be shown in the following section.

3.1. General Solution

The general solution to the Ricatti equation (38) is found from (Cooper *et al.*, 1995; Mielnik, 1984)

$$V_i^-(q^i) - E_i = \hat{W}_i^2 + \hat{W}_i' \equiv W_i^2 + W_i', \tag{39}$$

which by choosing

$$\hat{W}_i \equiv W_i + \frac{1}{y_i}, \tag{40}$$

leads to a Bernoulli equation for y_i,

$$y_i' - 2W_i \, y_i = 1, \tag{41}$$

whose solution is

$$y_i(q^i) = \frac{I_i + \gamma_i}{u_i^2}, \tag{42}$$

where $I_i(q^i) = \int_0^{q^i} u_i^2 dx$ and γ_i is the factorization parameter (γ_i plays the role of a "*time-like parameter*" in the evolution of the isospectral wavefunction).

By using Eq. (42), we may write Eq. (40) as

$$\hat{W}_i(q^i) = W_i + \frac{u_i^2}{I_i + \gamma_i}, \tag{43}$$

and, the entire family of bosonic potentials can be constructed from

$$\hat{V}_i^+(q^i, \gamma_i) - E_i = \hat{W}_i^2(q^i, \gamma_i) - \hat{W}_i'(q^i, \gamma_i), \tag{44}$$

or

$$\hat{V}_i^+(q^i, \gamma_i) = V_i^- - 2\hat{W}_i'$$

$$= V_i^+(q^i) - 4\frac{u_i u_i'}{I_i + \gamma_i} + 2\frac{u_i^4}{(I_i + \gamma_i)^2}, \qquad (45)$$

finally

$$\hat{u}_i \equiv g(\gamma_i)\frac{u_i}{I_i + \gamma_i}, \qquad (46)$$

is the isospectral solution of the Schrödinger equations (14, 15) for the new family of potentials (45), with the condition on the function $g(\gamma_i) = \sqrt{\gamma_i(\gamma_i + 1)}$, though in the limit

$$\gamma_i \to \infty \qquad g(\gamma_i) \to \gamma_i \qquad \text{and} \quad \hat{u}_i \to u_i. \qquad (47)$$

Considering the particular case for $\lambda < \sqrt{2}$ (which correspond to inflation), we plot the solutions in the variable $x = 6\alpha - \lambda\tilde{\phi}$ for the isospectral potential (45) (Figs. 1, 2 and 3) and the corresponding wavefunctions (46) (Figs. 4 and 5). The total WDW isospectral wave function has the following form

$$\Psi_{\text{iso}}(x, \tilde{y}; \gamma_1, \gamma_2) = \frac{\left[J_{i\eta}\left(\pm 2\sqrt{\beta}e^{x/2}\right) + J_{-i\eta}\left(\pm 2\sqrt{\beta}e^{x/2}\right)\right]}{I_1 + \gamma_1} \times \frac{\left[a_0 e^{i\frac{\eta\lambda}{2}\tilde{y}} + a_1 e^{-i\frac{\eta\lambda}{2}\tilde{y}}\right]}{I_2 + \gamma_2}. \qquad (48)$$
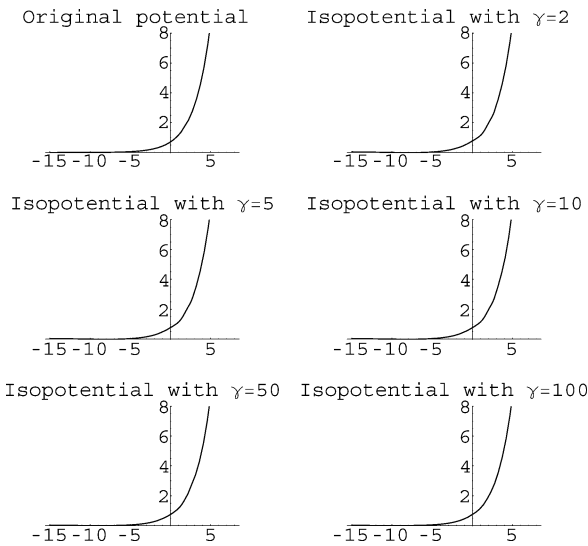

Fig. 1. Plot is of the original potential and the iso-potentials. At large scale, the original potential and the iso-potentials do not differ much.
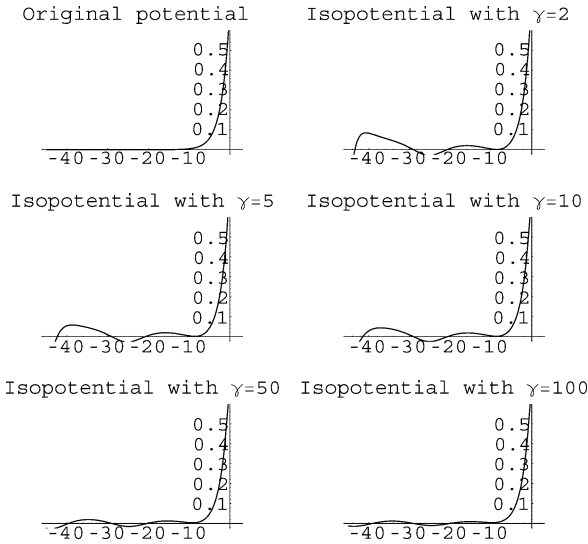
Original potential   Isopotential with $\gamma=2$

Isopotential with $\gamma=5$ Isopotential with $\gamma=10$

Isopotential with $\gamma=50$ Isopotential with $\gamma=100$

Fig. 2. At small scale the iso-potential have new structure for small values in the parameter γ, giving rise to a mechanism to end inflation that is not present in the original potential.

The γ_i parameters are included not for factorization reasons (these wave functions, in quantum cosmology are still non normalizable, except when a wave packet is constructed), but as decoherence parameter embodying a sort of quantum cosmological dissipation (or damping) distance. The wave function is highly oscillatory, but when the value of γ_i is increased, the iso-spectral wave function tends to the
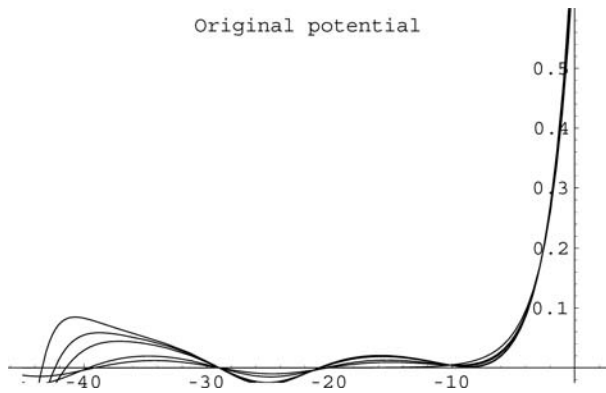
Original potential

Fig. 3. The corresponding changes in the structure for the isopotential, mainly for small values in the γ parameter.
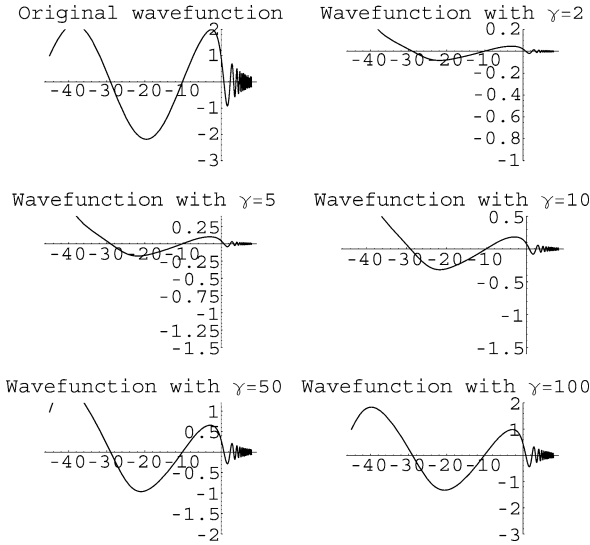
Fig. 4. The iso-wave function for different values for the γ parameter. The behavior for small values of γ can be seen. This different behavior can have effects in the quantum perturbations, and may modify structure formation in the large scale universe.

wave function of the original potential (this is similar to the transitory effects in quantum mechanics, i.e. probability density in a potential barrier). Unfortunately this parameter γ_i can not be used to go from one physical state to another, so it is not a time parameter in the usual sense. In other words, this parameter seems to play the role of a "supersymmetric time-like parameter" that gives the evolution from a supersymmetric state to the original state, and the usual behavior for the wave function is reached (see Figs. 3 and 5).

In the plots of the potentials, we see that the difference between the original potential and the isopotential are small at large scale (Fig. 1), but the corresponding wavefunctions present a drastically different behavior to a point that it vanishes for the limit $\gamma_i \to 0$ and recovers the original shape when $\gamma_i \to \infty$. But in all cases, the roots of the wave functions remain the same. This can be attributed to the different structure of the iso-spectral potential, as it appears in Figs. 2 and 3, in the sense that the amplitude of the wavefunctions corresponding to the iso-potentials in the regime $x < 0$, is reduced dramatically.

One expects that these effects should appear in the dynamics of our universe, in particular in the inflationary epoch. For inflation we need material with the unusual property of negative pressure (i. e. scalar fields), as already mentioned, the potential $V(\phi) = e^{-\lambda\phi}$ is one of the most studied. The standard approximation
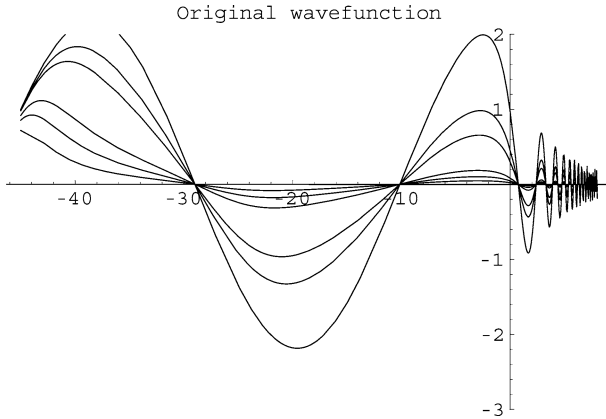
Original wavefunction

Fig. 5. This plot show how the iso-wave functions tend to original wave function when the parameter $\gamma \to \infty$.

technique for analyzing inflation is the slow-roll approximation given by the slow roll parameter $\epsilon = \frac{1}{2}(\frac{V'(\phi)}{V(\phi)})^2 \ll 1$ in units of $M_p = 1$, in particular for the exponential potential $\epsilon = \frac{1}{2}\lambda^2 \ll 1$, this means that once we fix λ to have inflation, there is no mechanism to end it (we can not violate the slow roll condition). For the iso-spectral potentials we can see from the plot that we have an oscillatory function superposed to the original potential and this oscillatory behavior leads to mechanism that ends inflation. To see this, we start by writing the slow roll parameter in terms of the potential $V(x)$, it takes the form $\epsilon = \frac{1}{2}a^2(\frac{V'(x)}{V(x)} + b)^2$, where $a = \frac{36-\lambda^2}{\lambda}$ and $b = \frac{36}{\lambda^2-6^2}$, for the original potential $\frac{V'(x)}{V(x)} = 1$. So again, once we fix the value of λ to have inflation there is no way to end it. For the iso-spectral potentials the approximate relation holds $\frac{V_{iso}(\phi)}{V(\phi)} \approx \frac{V_{iso}(x)}{V(x)}$ where $V_{iso}(x)$ is the iso-spectral potential, and again we may write the slow roll parameter in a similar way $\epsilon = \frac{1}{2}a^2(\frac{V'_{iso}(x)}{V_{iso}(x)} + b)^2$, where the constants a and b depend at most on λ, now in contrast with the original potential, once we fix the value of λ for inflation, the ratio $\frac{V'_{iso}(x)}{V_{iso}(x)}$ can be as large as we want. We can see this from the plots of the iso-potentials. If γ_i is small, the potential is oscillatory and has several roots. In particular near one of the roots x_i $V'_{iso} \neq 0$, but $V(x) \to 0$ as $x \to x_i$, that is, in the neighborhood of x_i, $\frac{V'_{iso}(x)}{V(x)}$ is large, violating the slow-roll conditions, this gives the mechanism to end inflation!. Moreover, once inflation has ended the iso-potentials have several minima, where the scalar field can oscillate and start the reheating process. Finally there is a non zero probability that the scalar field may tunnel out of this minimum and give start another inflationary epoch.

4. CONCLUSIONS AND OUTLOOK

In this paper we have applied the factorization method to inflationary cosmology, we started with a scalar field model coupled to and FRW model and an exponential potential $V(\phi) = V_0 e^{-\lambda\phi}$. We have shown that in the variables x and y we can reproduce the known result, that for $\lambda < 2$ and get power law inflation. This new set of variables is important to apply the factorization approach of SUSY-QM, from which we have obtained a complete family of iso-spectral potentials and their corresponding wave functions. We have shown in the plots, that we obtain the original potential and wave function in the limit $\gamma_i \to \infty$. Out of this limit we have a large class of potentials that have the same quantum spectrum (this can clearly be seen in Fig. 5). Therefore all of these potentials describe the same quantum system. These iso-potentials have a very similar behavior for large scales, so at the classical level we won't see different dynamics. The drastic differences in the iso-wave functions and the iso-potentials, have an impact on the the dynamics of the universe. We have seen that this family of iso-potentials, have a natural mechanism to end inflation, and present the possibility of new inflationary epochs. We have found that the γ_i parameter plays the role of a decoherence parameter that embodies a type of quantum cosmological damping distance. In other words, it plays the role of a supersymmetric time-like parameter. This procedure can be applied to other models like, tachyon driven cosmology (Garcia-Compean *et al.*, 2005), or to toy models of the landscape of string theory (Kobakhidze and Mersini-Houghton, 2004). These results may point to the existence of other potentials with the same eigenvalue spectrum, so that in principle the landscape may be falsifiable. These ideas are being explored and will be presented elsewhere.

ACKNOWLEDGMENTS

REFERENCES

Bagchi, B. K. (2001). *Supersymmetry in Quantum and Classical Mechanics*, Chapman & Hall, New York.

Bagrov, B. G. and Samsonov, B. F. (1995). *Theoretical Mathematical Physics* **104**, 356.

Cooper, F. Khare, A., and Sukhatme, U. (1995). *Physics Repert* **251**, 267.

Chimento, L. P. and Jakubi, A. S. (1996). *International Journal Modern Physics D* **5**, 71.

Darboux, G. (1882). *Comptes Rendus Hebdomadaires des Seances de l'Academie des Sciences (Paris)* **94**, 1456.

Douglas, M. R. (2003). *Journal of High Energy Physics* **305**, 046.

Fernández, D. J. (1984). *Letters Mathamatical Physics* **8**, 337.

Garcia-Compean, H., García-Jimenez, G., Obregón, O., and Ramírez, C. (2005). *Physical Review D* **71**, 063517.

Gelfand, I. M. and Levitan, B. M. (1955). *American Mathematical Society Translation* **1**, 253.

Gorini, V., Kamenshchik, A. Y., Moschella, U., and Pasquier, V. (2004). *Physical Review D* **69**, 123512.

Gradshteyn, I. S. and Ryzhik, I. M. (1980). *Table of Integrals, Series, and Products*, Academic Press, pp. 773–774.

Guzmán, W., Sabido, M., Socorro, J., and Arturo Urena López, L. *Scalar potential out of canonical quantum cosmology*, gr-qc/0506041.

Ince, E. L. (1926). *Ordinary Differential Equations*, Dover, New York.

Junker, G. (1996). *Supersymmetric Methods in Quantum and Statistical Physics*, Springer, Berlin.

Kiefer, C. (1988). *Physical Review D* **38**, 1761.

Kiefer, C. (1990). *Nuclear Physics B* **341**, 273.

Kobakhidze, A. and Mersini-Houghton, L. hep-th/0410213.

Mersini-Houghton, L. (2005). *Classical and Quantum Gravity* **22**, 3481–3490.

Mielnik, B. (1984). *Journal Mathematical Physics* **25**, 3387.

Nieto, M. M. (1984). *Physics Letters B* **145**, 208.

Pappademos, J., Sukhatme, U., and Pagnamenta, A. (1993). *Physical Review A* **48**, 3525.

Rosu, H. and Socorro, J. (1996). *Physics Letters A* **223**, 28.

Rosu, H. and Socorro, J. (1998). *Il Nuovo Cimento B* **113**, 683.

Ryan, M. Jr. (1972). *Hamiltonian Cosmology*, Springer Verlag.

Samsonov, B. F. and Suzko, A. A. *Discrete supersymmetries of the Schrödinger equation and non-local exactly solvable potential*, quant-ph/0301109.

Sen, A. (2002). *Modern Physics Letters A* **17**, 1797.

Sen, A. (2003). *International Journal Modern Physics A* **18**, 4869.

Socorro, J. Reyes, M. A., and Gelbert, F. A. (2003). *Physics Letters A* **313**, 338-342.

Susskind, L. hep-th/0302219.